Available online at www.sciencedirect.com
ScienceDirect
Procedia Engineering 12 (2011) 184–190
Engineering Procedia
the 2011 SREE Conference on Engineering Modelling and Simulation

Applied Research on Improved Fuzzy Chance-constrained Model in Engineering Project Comparison and Selection

Peng Zuojun[a], Chen Yuhong[b], Shi Lei[a,b, a*]

[a]Faculty of Construction Management and Real Estate,Chongqing University,Chongqing 400045,China
[b]Airport New Town Administration of Chongqing,Chongqing 401120,China

Abstract

Subjective choice rises of base efficiency are reduced by multi-target programming theory of fuzzy chance constraints. The minimum value of confidence is found which meet Constraint Conditions of every Decision Making Units and the efficiency of the method is verified by an example. The results indicate that principle of the method is correct, improved Fuzzy Chance Constrained DEA Model is effective supplement of original model which widen decision maker's choice range and make assessment overall.

Keywords: Fuzzy chance constraints; Date envelopment analysis; Base efficiency

1. Introduction

DEA method[1] is a new system analysis method developed by A.Charnes and W.W.Cooper, the famous American experts on operations research, on the basis of "relative efficiency evaluation" concept. This method mainly applies the mathematical programming method, and uses the observed effective sample data to make the evaluation of production effectiveness on DMU (Decision Making Units). In traditional DEA evaluation, all input/output data are required to be accurate, however in the reality, because of the influence of some uncertainty factors, such as defects of statistical methods, insufficiencies of information source, and randomness of economic phenomena, the data cannot be accurate which makes the use of traditional DEA method to evaluate the data become very sensitive. In addition, the traditional

* Corresponding author. Tel.: 008613637783756; fax: 008602365926083.
E-mail address: cqshilei@126.com.

1877–7058 © 2011 Published by Elsevier Ltd. Open access under CC BY-NC-ND license.
doi:10.1016/j.proeng.2011.05.029

DEA method measures the efficiency of DUM on the basis of base efficiency for 1, and in some adverse circumstances, the uncertain data may lead to the result that the decision-making cannot meet the constraints in a certain extent. Therefore, policy makers hope the possibility of proper constraints is not less than a certain confidence levels [2], therefore, from the angle of policy makers' satisfaction, it greatly broaden selection and evaluation of the project.

Based on the defects of traditional DEA method, fuzzy DEA model comes into being. Li Guangjin[3] and Wu Wenjiang[4] take the input-output elements as the fuzzy number, and establish the DEA model according to the fuzzy number algorithms. Using fuzzy decision theory, Konstantions Triantis[5] put forward three-step method of DEA evaluation under fuzzy environment. Pei Junguo[6] use the input-output elements and constraint blurring to work out the center efficiency of DMU and its spread around, and get the fuzzy efficiency of DMU. Since the time that Land, Lovell[7] used the chance constraints programming to establish the efficiency of a given observation set, Olesen[8] proposed the chance constrained DEA model, and made segment Linear envelope for the observed random input and output confidence. Cooper[9] draw from the "satisfaction" concept from the behavior theory of management and psychology proposed by Simon to establish the chance constraints probability model. Zhang Maoqin[10] establishes the fuzzy chance constrained model which is based on the fuzzy chance constraints programming proposed by Liu and Iwamura[11,12]. The model processes the fuzziness and randomness of input-output data, introduces base efficiency to replace ideal efficiency 1 from the angle of the satisfaction of policy makers, deals with the uncertainty of the model through the confidence, and offers a new thought for the DEA model. However, because the base efficiency is determined by people subjective, it will increase the risk of the accuracy the model evaluation inevitably, however, by using the multi-target programming theory of fuzzy chance constraints to improve it and seek the minimum value of confidence meeting the constraints of each DMU, and ultimately reduce the influence of human factors by mathematical methods.

2. Fuzzy chance constrained DEA model

2.1. Theory model

Let a total of n-DMU, each DMU consists of m inputs and s output, then[13]:

$$\begin{cases} \min E_{jo} = pos\left\{\eta_0 V^T \tilde{X}_0 \geq U^T \tilde{Y}_0\right\} \\ \text{s.t.} pos\left\{\eta_j V^T \tilde{X}_j \geq U^T \tilde{Y}_j\right\} \geq \beta_j \\ U \geq 0, V \geq 0; j = 1, 2, \cdots, n \end{cases} \quad (1)$$

η_j means the maximum efficiency that policy-makers hope the DMU_j can achieve and it instead of the ideal efficiency 1. pos｛•｝is the Measurement of P which means the possibility of the event in｛•｝and it is not common-sense probability while it is the possible theory based on Zadeh which can estimate the degree to which the membership functions of the two targets are comparable. βj is given by the policy maker prior which means the confidence that the event in pos｛•｝has to achieve at least.

$\tilde{X}_j = (\tilde{x}_{1j}, \tilde{x}_{2j}, \cdots, \tilde{x}_{mj})^T, \tilde{x}_{ij} = (x_{ij} : c_{ij}{}^l, c_{ij}{}^r)$, x_{ij} is the principal value of $\tilde{x}_{ij}$; $c_{ij}{}^l, c_{ij}{}^r$ is the arounddiffusion of $\tilde{x}_{ij}$ respectively. It is a special fuzzy number of L-R model and other kinds of fuzzy number can be transformed into a L-R model and the proper is the convenience of calculation. The meaning of symbols in $\tilde{Y}_j = (\tilde{y}_{1j}, \tilde{y}_{2j}, \cdots, \tilde{y}_{sj})^T$, $\tilde{y}_{rj} = (y_{rj} : d_{rj}{}^{l}, d_{rj}{}^{r})$ is similar to the above ones.

i=1,2, • • • ,m;r=1,2, • • • ,s;j=1,2, • • • ,n。

The model expounds that under the opportunity meeting the constraint conditions, if the ratio of the fuzzy input and fuzzy output of DMU is not greater than maximum efficiency that policy makers expected to achieve, then the confidence levels of this event is as small as possible.

2.2. Solutions to the model

The objective function of theory DEA model is nonlinear, and the introduction of variable θ can turned the objective function in the model into a linear function. While the constraint conditions of Model (1) is with uncertainty, by literature[14] the lemma of 2.1 and 2.2, it is translated into certainty equivalent constraints, and by the Charnes-cooper transform finally get the following linear programming model.

$$\begin{cases} \min \mathrm{E}_{j0} = \theta, \quad \text{s.t.} \\ \theta \geq 1 + \eta_0 \omega^{\mathrm{T}} \mathrm{X}_0 \text{-} \mu^{\mathrm{T}} Y_0 \\ \eta_j \omega^{\mathrm{T}} [\mathrm{X}_j + (1-\beta_j) c_j^r] \geq \mu^{\mathrm{T}} [Y_j - (1-\beta_j) d_j^l] \\ \eta_0 \omega^{\mathrm{T}} c_0^r + \mu^{\mathrm{T}} d_0^l = 1 \\ \omega \geq 0, \mu \geq 0, \theta \geq 0, \end{cases} \tag{2}$$

If and only if θ = β_0 , DMU0 is fuzzy DEA effective when confidence is θ compared with the base efficiency η_0 .

3. The improvement of Fuzzy chance constrained DEA model

Since the base efficiency η_j is given by decision-makers in advance according to their own hope target, so in the evaluation process some programs may become infeasible because of the policy makers' subjective factors. In order to reduce the risk caused by subjective factors, by changing the one-to-one relationship of h_j and DMU_j, a series of η_j is introduced into the objective function and DMU_0 relative base efficiency η_j gets the minimum confidence. Model changes from a single- target programming DEA model (1) into a multi-target programming DEA model, namely:

$$\begin{cases} \min E_{jo} = pos\left\{\left\{\eta_1 V^T \tilde{X}_0 \geq U^T \tilde{Y}_0\right\}, \left\{\eta_2 V^T \tilde{X}_0 \geq U^T \tilde{Y}_0\right\}, \cdots, \left\{\eta_n V^T \tilde{X}_0 \geq U^T \tilde{Y}_0\right\}\right\} \\ s.t. pos\left\{\eta_j V^T \tilde{X}_j \geq U^T \tilde{Y}_j\right\} \geq \beta_j \\ U \geq 0, V \geq 0; j = 1, 2, \cdots, n \end{cases} \tag{3}$$

The multi-target programming can be converted into a single- target programming through Zimmermann's fuzzy algorithm [14], and define the membership function of goal i, $\lambda_i = \dfrac{1-\sigma_i}{1-\bar{\sigma}_i}$, $\bar{\sigma}_i \le \sigma_i \le 1$.The Solving process of $\bar{\sigma}_i$ is the same with the fuzzy chance constrained DEA model solution, and we get the following through the transformation and sorting:

$$\begin{cases} \min E_{j0} = \bar{\sigma}_i, \quad \text{s.t.} \\ \sigma \ge 1+\eta_j\omega^T X_0 - \mu^T Y_0 \\ \eta_j\omega^T[X_j + (1-\beta_j)c_j^r] \ge \mu^T[Y_j - (1-\beta_j)d_j^l] \\ \eta_j\omega^T c_0^r + \mu^T d_0^l = 1 \\ \omega \ge 0, \mu \ge 0, \theta \ge 0, \end{cases} \qquad (4)$$

The difference from (2) lies in the base efficiency η_i, so that the optimized solution of each DMUj are based on their own and other base efficiency. The equivalent linear programming derived by Zimmermann fuzzy algorithm is as followings [15]:

$$\begin{cases} \max \lambda, \text{s.t.} \\ \lambda_i = \dfrac{1-\sigma_i}{1-\bar{\sigma}_i} \\ 0 \le \lambda \le 1 \\ (\mu, \nu, \sigma) \in R; i=1,2\cdots n \end{cases} \qquad (5)$$

R is the feasible region of programming (3), and the proof of the feasible solution can be found in the literature [15]. If the optimal solution of programming (5) is $\lambda^*, \sigma^*, \mu^*, v^*$, the confidence of multi-target programming (3) is $\alpha = \min_i \sigma_i^*$,and $\sigma_i^* \ge \beta_{i0}$. Let $\alpha = \sigma_k{}^*$, then the corresponding base efficiency is η_k. If and only if a$=\beta_0$, DMU_0 is fuzzy DEA effective when confidence is a compared with the base efficiency η_k.

4. Case study

Previous experience proved that DEA method has the advantage of applying in project evaluation and optimization [16]. A company proposes to build an ethylene plant, through the technical and economic data research of large scale production company with the same and different size, it gets even the same size manufacturing enterprises can lead to input and output difference, because of its internal management and external market conditions and other factors, so the basic data in table 1 is indicated by the processing and analysis of the original data after research by L-R type fuzzy numbers. Considering the risks and personal preferences, policymakers choose the corresponding base efficiency and confidence, and its corresponding data is also listed in table 1. Hereby after the evaluation of the project, appropriate proposed ethylene production scale will be chosen.

Table 1 Technical and economic foundation data of different scale ethylene production enterprise

Index		DMU1	DMU2	DMU3	DMU4
Scale	(wan t • a^{-1})	100	150	250	300
Input index	Unit ability investment (yuan • t^{-1})	(4835:176, 210)	(4438:120, 250)	(4230:173, 198)	(4198:200, 292)
	Unit ability cost (yuan • t^{-1})	(1732:82, 78)	(1622:40, 87)	(1553:39, 49)	(1526:35, 85)
Output index	Years of project operation (year)	(28.1:2.3, 1.9)	(30.2:1.9, 2.1)	(31.4:1.1, 2.6)	(31.7:2.9, 2.5)
	Financial internal rate of return (%)	(14.51:2.6, 4.2)	(16.28:3.2, 1.8)	(18.5:1.6, 2.3)	(18.9:2.5, 3.8)
	η_i	0.95	0.96	0.97	0.99
	β_j	0.95	0.97	0.95	0.99

Note: Investment of unit ability= total investment / production scale; Cost of unit ability= total cost / production scale.

According to type (2) fuzzy chance constraints DEA model, the solution results are shown in Table 2.

Table 2. The computational results of fuzzy chance constrained DEA model before the improvement

Index	DMU1	DMU2	DMU3	DMU4
θ	1	1	0.963	0.99
η_j	0.95	0.96	0.97	0.99
β_j	0.95	0.97	0.95	0.99

Through calculation, DMU1 is fuzzy negation DEA effective when confidence is 1 compared with the base efficiency 0.95. DMU2 and DMU3 is the same with DMU1. DMU4 is fuzzy DEA effective when confidence is 0.99 compared with the base efficiency 0.99.

According to type (4), the solution results are shown in Table 3.

Table 3 The calculation results of fuzzy chance constrained DEA model based on different base efficiency

	$\overline{\sigma}_1$	$\overline{\sigma}_2$	$\overline{\sigma}_3$	$\overline{\sigma}_4$
DMU1	1	1	1	1
DMU2	1	1	1	1
DMU3	0.6907	0.95	0.963	1
DMU4	0.7113	0.7818	0.8517	0.99
η_j	0.95	0.96	0.97	0.99

Using the linear programming (5) type to get the solution, and because $\alpha = \min_i \sigma_i^*$, $\sigma_i^* \geq \beta_{i0}$, we can the calculation results of fuzzy chance constraints DEA model after the improvement, and it is shown in table 4.

Table 4 the calculation results of fuzzy chance constraints DEA model after the improvement

Index	DMU1	DMU2	DMU3	DMU4
a	1	1	0.95	0.99
η_j	$\forall\eta$	$\forall\eta$	0.96	0.99
β_j	0.95	0.97	0.95	0.99

The comparison of calculated results in the table 2 and table 4 shows that before the improvement, DMU4 is fuzzy DEA effective when confidence is 0.99 compared with the base efficiency 0.99, while DMU3 is fuzzy negation DEA effective when confidence is 0.963 compared with the base efficiency 0.97; after the improvement, DMU3 is fuzzy DEA effective when confidence is 0.95 compared with the base efficiency 0.96. Therefore, the policy makers can select the corresponding scale according to their risk preferences and economy ability. If the investors have abundant funds, they can firstly choose 300 tons/year ethylene production scale; if the capital is limited, they can also be consider to choose 2.5 million tons/year ethylene production scale, after all its confidence is 95 percent.

5. Conclusion

The improved fuzzy chance constrained DEA model is the effective complement to fuzzy chance constrained DEA original model, broaden the chances of optional project in reality when policymakers face all kinds of difficulties, reduce the risk of base efficiency choices, and make evaluation results more comprehensive, and meet realistic decision needs.

References

[1] Charnes A,Cooper W.W,Rhodes E.Measuring the Efficiency of Decision Making Units.*European Journal of Operational Research*,1985,(2):6-7.

[2]Zhang Maoqin; LI Guangjin. A Fuzzy Chance Constrained DEA Model With Fuzzy Triangular Factors. *Mathematics In Practice and Theory*, 2004,（2）：42.

[3]LI Guangjin A DEA Model With Fuzzy Triangular Factors.*Journal of Systems Engineering*，1996，（2）：37－44.

[4]Wu Wenjiang, Chen Ying .Solving an Estimation Problem on Input or Output with Fuzzy Number by DEA. *Systems Enging-theory Methodology Application*, 2000，(4):330-334.

[5]Konstantions Triantis,Olivier Girod.Amathematical programming approach for measuring technical efficiency in fuzzy environment.*Journal of Productivity Analysis*,1998,(10):85-102.

[6]Peijun Guo,Hideo Tanaka.Fuzzy DEA:A perceptual evaluation method. *Fuzzy Sets and Systems*,2001,(1):149-160.

[7]Land K C,Lovell C A K.Thodes S.Chance-constrained Data Envelopment Analysis.*Managerial Decision Economics*,1993,(14):541-554.

[8]Olesen O.B and N C.Petersen. Chance constrained efficiency evaluation. *Management Science*,1995,(41):442-457.

[9]W.W.Cooper. Chance constrained programming formulations for stochastic characterizations of efficiency and dominance in DEA. *Jounal of Productivity Analysis*,1998,(9):53-79.

[10]Zhang Maoqin. Research on Chance-constrained DEA Models with Fuzzy Factors [D].Si Chuan: Sichuan University, 2004

[11]Baoding Liu,Kakuzo Iwamura. Fuzzy programming with fuzzy decisions and fuzzy simulation-based genetic algorithm. *Fuzzy Sets and Systems*,2001,(2):253-262.

[12]Baoding Liu,Kakuzo Iwamura.A note on chance constrained programming with fuzzy coefficients. *Fuzzy Sets and Systems*,1998,100(1-3):229-233.

[13]Zeng Yunxiang, Wu Yuhua.L-R model Interval DEA model and its Transformation. *System Engineering*，2000，（3）：60-61.

[14]Li Rongjun. *Fuzzy multi-criteria decision theory and application*. Beijing: science press，2002.

[15]Guo Minglei. Fuzzy Chance Constrained Multi-objective DEA Model Research[D]. Si Chuan: Sichuan University,2006

[16]Hou Huafeng, Zhang Xu. Application of DEA Method on Petroleum enterprise economic benefit evaluation. *The theory method and implication of system engineering*，2000，（3）：252-257.